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QUARTERLY PROGRESS REPORT No. 4 Under Contract NObsr-95181

For the Period 1 May - 31 July 1967

Resulting from Research done Under
NAVAL SHIP SYSTEMS COMMAND
CONTRACT NObsr-95181
Project Serial No. SF1010316, Task 8641

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I. CEPSTRUM ANALYSIS

(J. A. Shooter, C. J. Webb)

It was pointed out in the previous progress report[1] that the techniques for computing a Cepstrum have certain shortcomings in a mathematical sense. These restrictions were placed on the technique by certain approximations and assumptions that were made concerning the relative amplitudes of the original signal and echoes. This past quarter, effort was made to derive alternate techniques for computing Cepstrum, using as few mathematical assumptions as possible.

In the following discussions, the standard method for computing a Cepstrum as defined by B. P. Bogert *et al*,[2] will be referred to as Cepstrum. The alternate methods will be defined as derived.

A. First Alternate for Cepstrum Analysis (DRL A-I)

1. Mathematical Description for a Single Echo

Consider the time series

$$z(t) = y(t) + Ay(t - \tau) \quad , \tag{1}$$

[1] Defense Research Laboratory, The University of Texas at Austin, Quarterly Progress Report No. 3 under Naval Ship Systems Command Contract NObsr-95181, 1 February - 30 April 1967, pp. 2-7.

[2] B. P. Bogert, M. J. R. Healy, and J. W. Tukey, in "Proceedings of the Symposium on Time Series Analysis," ed. M. Rosenblatt (John Wiley and Sons, Inc., New York, 1963), Chap. 15, pp. 209-243.

consisting of the signal $y(t)$ together with the same signal delayed in time by the amount τ and damped in amplitude by the factor A.

The following notation will be used throughout:

$f_z(\omega)$ = Fourier transform of $z(t)$,

$f_y(\omega)$ = Fourier transform of $y(t)$,

$\Phi\ (\omega)$ = power spectrum of $z(t)$,

$F\ (\omega)$ = power spectrum of $y(t)$,

ω = angular frequency in rad/sec.

Then, from Eq. (1),

$$\Phi(\omega) = f_z(\omega) \cdot f_z^*(\omega) = f_y(\omega) \cdot f_y^*(\omega)(1 + Ae^{i\omega\tau})(1 + Ae^{-i\omega\tau}) \quad , (2)$$

where the asterisk denotes complex conjugation. Therefore,

$$\Phi(\omega) = F(\omega)(1 + 2A \cos \omega\tau + A^2) \quad . \quad (3)$$

For those values of ω for which $\Phi(\omega) \neq 0$, $F(\omega) \neq 0$, $\Phi'(\omega)$ and $F'(\omega)$ exist;[3]

$$\frac{\Phi'(\omega)}{\Phi(\omega)} = \frac{F'(\omega)}{F(\omega)} - \frac{\tau \sin \omega\tau}{\left[(A^2 + 1)/2A\right] + \cos \omega\tau} \quad . \quad (4)$$

Thus, the logarithmic derivative of the power spectrum of the composite signal, $z(t)$, is the logarithmic derivative of the original signal, $y(t)$, with an added periodic function,

[3] The prime denotes a derivative with respect to the dependent variable.

$$\frac{-\tau \sin \omega\tau}{\left[(A^2 + 1)/2A\right] + \cos \omega\tau} .$$

This added term is periodic in $\omega/2\pi$ with period $1/\tau$. Hence, if $\frac{F'(\omega)}{F(\omega)}$ is relatively flat, this additional term should appear as a "ripple" of period $1/\tau$ superimposed on the log derivative of $F(\omega)$.

If a spectral analysis of $\frac{\Phi'(\omega)}{\Phi(\omega)}$ is now performed, and if $\frac{F'(\omega)}{F(\omega)}$ is relatively flat, the component with frequency $1/\tau$ should contribute significantly. Specifically, examining the power spectrum, $P(\alpha)$, of $\frac{\Phi'(\omega)}{\Phi(\omega)}$, a peak at $\alpha = \tau$ is expected.

In contrast to Cepstrum, this approach, to be referred to as DRL A-I, requires no linear approximation to a logarithm function. Also, no restrictions have been placed on the relative amplitudes of the delayed and undelayed signals as is necessary, at least theoretically, in the case of Cepstrum.

2. Practical Comparison of Cepstrum and DRL A-I

The alternate method for computing a Cepstrum, DRL A-I, is not more difficult to implement on the digital computer than the standard Cepstrum technique. The logarithmic derivative can be computed by a standard difference scheme.

As an example of the methods, consider the signal $z(t)$ to contain only one echo as described by the relation

$$z(t) = \sin[2\pi ft] + H(t - \tau) \sin[2\pi f(t - \tau)] \quad , \qquad (5)$$

where the amplitude of the echo is given as

$$H(t) = \begin{cases} 0, \ t < 0 & , \\ 1, \ t \geq 1 & . \end{cases} \tag{6}$$

The sample length will be 2.0 sec with a bandwidth from 50 to 150 Hz. The sampling rate is 1000 Hz.

Figure 1 shows the time function given by Eq. (5) processed by both the Cepstrum and DRL A-I techniques. As expected, both methods give the same results for this simple model.

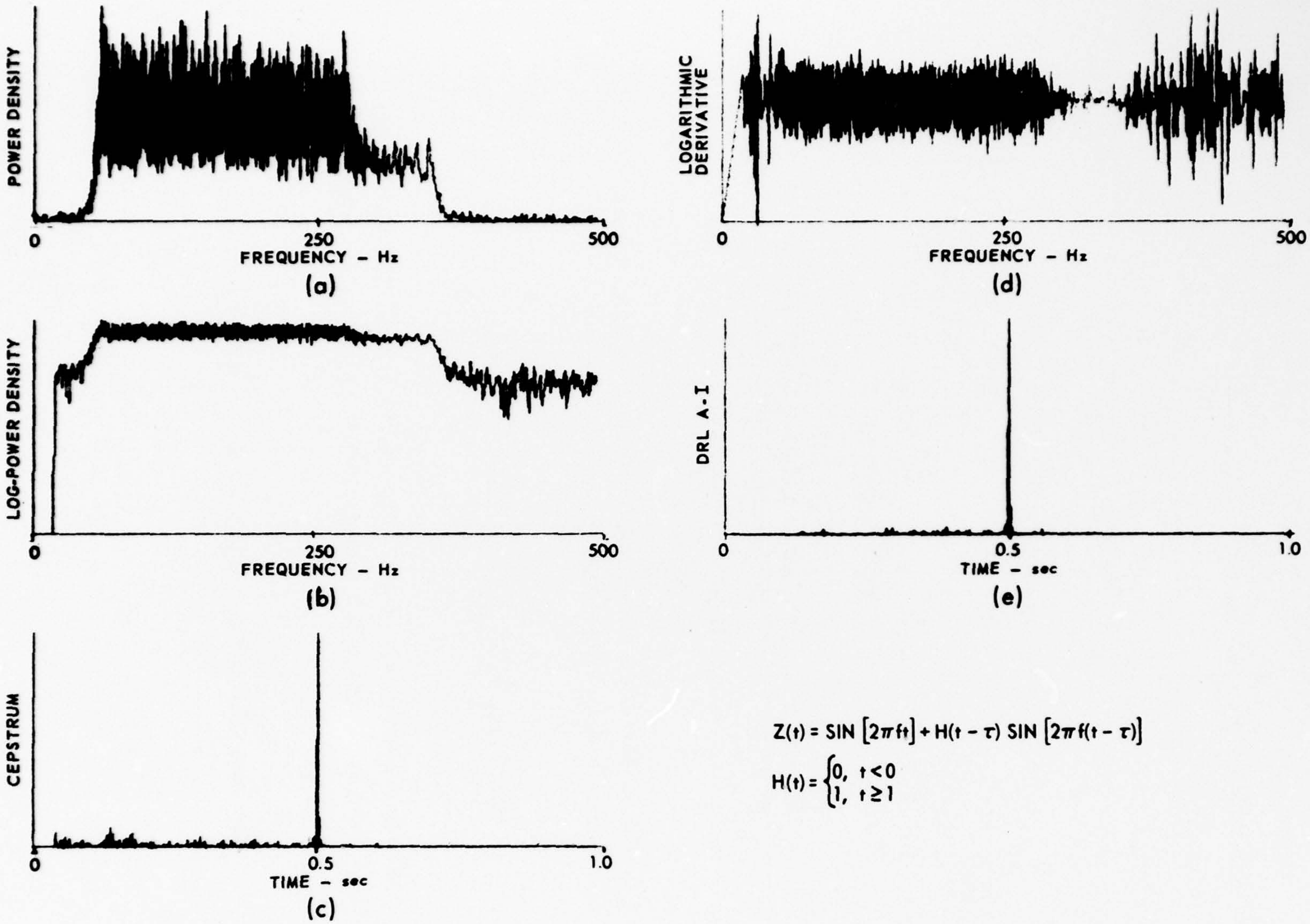
3. <u>Multiple Echoes</u>

Consider first the case of two time delays or echoes given as

$$z(t) = Y(t) + A_1 y(t - \tau_1) + A_2 y(t - \tau_2) \quad . \tag{7}$$

In this case,

$$f_z(\omega) = f_y(\omega) \left[1 + A_1 e^{-i\omega\tau_1} + A_2 e^{-i\omega\tau_2} \right] \quad , \text{ and} \tag{8}$$



$$Z(t) = \mathrm{SIN}\left[2\pi f t\right] + H(t-\tau)\ \mathrm{SIN}\left[2\pi f(t-\tau)\right]$$

$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 1 \end{cases}$$

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FIGURE 1
COMPARISON OF CEPSTRUM AND DRL A-I ANALYSIS

$$\Phi(\omega) = F(\omega)\left|1 + A_1 e^{-i\omega\tau_1} + A_2 e^{-i\omega\tau_2}\right|^2$$

$$= F(\omega)\left\{1 + A_1^2 + A_2^2 + 2A_1 \cos \omega\tau_2\right. \tag{9}$$

$$\left. + 2a_2 \cos \omega\tau_2 + 2A_1A_2 \cos \omega(\tau_1 - \tau_2)\right\} \quad .$$

Thus,

$$\frac{\Phi'(\omega)}{\Phi(\omega)} = \frac{F'(\omega)}{F(\omega)}$$

$$- \frac{A_1\tau_1 \sin \omega\tau_1 + A_2\tau_2 \sin \omega\tau_2 + A_1A_2 (\tau_1 - \tau_2) \sin \omega(\tau_1 - \tau_2)}{\frac{1 + A_1^2 + A_2^2}{2} + A_1 \cos \omega\tau_1 + A_2 \cos \omega\tau_2 + A_1A_2 \cos \omega(\tau_1 - \tau_2)} \quad . \tag{10}$$

If A_1 and A_2 are small in modulus compared to unity, and the higher order terms in A_1A_2 are negligible, then

$$\frac{\Phi'(\omega)}{\Phi(\omega)} \cong \frac{F'(\omega)}{F(\omega)} - \frac{A_1\tau_1 \sin \omega\tau_1 + A_2\tau_2 \sin \omega\tau_2}{1/2 + A_1 \cos \omega\tau_1 + A_2 \cos \omega\tau_2} \quad . \tag{11}$$

The numerator and denominator of the second term on the right side of Eq. (11) each consist of terms periodic in $\omega/2\tau$ with periods $1/\tau_1$ and $1/\tau_2$.

In the case of N delays, it is simple to see that

$$\frac{\Phi'(\omega)}{\Phi(\omega)} \cong \frac{F'(\omega)}{F(\omega)} - \frac{\sum_{k=1}^{N} A_k \tau_k \sin \omega\tau_1}{1/2 + \sum_{k=1}^{N} A_k \cos \omega\tau_k} \quad . \tag{12}$$

It is of interest to note that the simplification recommended (ignoring the higher order terms in the A's) can be made either before or after taking the logarithmic derivative; i.e., either in Eq. (9) or in Eq. (10). Equation (11) is the result regardless.

4. Single Echo With Noise

If we consider a background noise in addition to the signal, the simple model described by Eq. (1) becomes

$$z(t) = n(t) + y(t) + Ay(t - \tau) \quad . \tag{13}$$

In this case

$$\begin{aligned} \Phi_z(\omega) &= F_n(\omega) + 2\mathrm{Re}\left\{f_n(\omega)\, f_y^{\,*}(\omega)\, (1 + Ae^{i\omega\tau})\right\} \\ &\quad + F_y(\omega)\left[1 + A^2 + 2A\cos\omega\tau\right] \quad , \end{aligned} \tag{14}$$

where the asterisk indicates complex conjugation, Re denotes the real component of the expression in brackets, and the other notation is:

$f_n(\omega)$ = Fourier transform of n(t),

$f_y(\omega)$ = Fourier transform of y(t),

$F_n(\omega)$ = Power spectrum of n(t),

$F_y(\omega)$ = Power spectrum of y(t).

Equation (14) shows the degree to which noise can compound the problem. However, if n(t) is a white noise source--clearly the most desirable situation--then $F_n(\omega)$ is flat. $F_n'(\omega)$ therefore vanishes and the $F_n(\omega)$ term makes no contribution to the logarithmic derivative of $\Phi(\omega)$. Note that the second term on the right hand side of Eq. (14) will tend to zero with increasing values of signal to noise ratio. Thus, in the case of a white noise source and high signal-to-noise ratio, the logarithmic derivative of $\Phi_z(\omega)$ [$\Phi_z(\omega)$ as given by Eq. (14)] can be made to approximate Eq. (4).

B. Second Alternate for Cepstrum Analysis (DRL A-II)

From the previous development, it is clear that a signal, y(t), with an optimally flat power spectrum is the most desirable for additional processing such as Cepstrum or DRL A-I. For example, if the signal is a sine pulse, the frequency is modulated to achieve a power spectrum that is approximately flat.

A flat power spectrum can be obtained by a more judicious choice of the signal y(t). Suppose that y(t) is a signal whose Fourier transform, $f_y(\omega)$, possesses the properties:

$$f_y(\omega) = \begin{cases} B, \; 0 \leq \omega \leq \omega_o \; , \\ 0, \; \omega_o < \omega \; , \end{cases} \tag{15}$$

and $f_y(\omega)$ is real.

For a signal composed of y(t) and a single echo, such as

$$z(t) = y(t) + y(t - \tau) \quad , \tag{16}$$

the Fourier transform is

$$f_z(\omega) = \begin{cases} B\left[1 + Ae^{-i\omega\tau}\right], & 0 \le \omega \le \omega_o \quad , \\ 0 & , \omega_o < \omega \quad . \end{cases} \tag{17}$$

The Imaginary component of $f_z(\omega)$ is

$$\text{Im } f_z(\omega) = -AB \sin \omega\tau \quad . \tag{18}$$

Thus, Im $f_z(\omega)$ consists of a single periodic component, and the power spectrum of Im $f_z(\omega)$ is a delta function of time.

For the case of multiple echoes or N time delays, it can be shown that the result will be N delta functions--one delta function at each of the points in time τ_k, k = 1, 2, ..., N.

The advantages of this second alternate method lie in its obvious simplicity and short computation time. However, whereas Cepstrum and DRL A-I are designed for use with a more general type signal, this last method requires a signal with very special properties. One example of a signal of this type is

$$y(t) = \frac{\sin(\omega t)}{\omega t} \quad , \tag{19}$$

which has a rectangular shaped Fourier transform.

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II. DIGITAL FILTERS
(G. E. Ellis, H. W. Pitman)

A. Nonrecursive Filters

For a linear filter described by the weighting function $h(t)$, the output of the system $\hat{u}(t)$, can be related to the system input, $u(t)$, over finite time limits T_L, T_u by the relation

$$\hat{u}(t) = \int_{-T_L}^{T_u} h(\tau)s(t - \tau)d\tau \quad . \tag{20}$$

For discrete or sampled data, this expression can be given as

$$\hat{u}_m = \sum_{n=-L}^{u} h_n \, u_{n+m} \quad , \tag{21}$$

where h_n are the filter weights, u_{n+m} are the input samples, and $\hat{u}_m$ are output samples. The spacings n and m are determined by the sampling rate needed to prevent aliasing.

For infinite time limits $(-\infty \leq n \leq \infty)$ the frequency function, $H(\omega)$, of the filter together with the weighting function, continuous or discrete, form particular Fourier transform pairs. In the case of finite time limits, it is desirable to choose a finite set of weights, h_n, whose frequency is a least square fit to a designed frequency function.

A class of filters having a frequency response function of unity gain and zero phase shift in the passband with a pth order roll-off between the passband, $(\omega \leq \omega_c)$, and the rejection bonds, $(\omega \geq \omega_T)$, is given in Fig. 2.

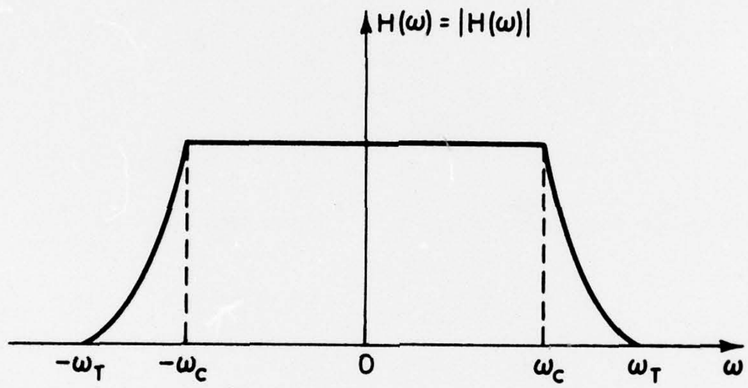

FIGURE 2
FREQUENCY FUNCTION FOR A CLASS OF LINEAR FILTERS
$\omega_c = 2\pi f_c$ = CUTOFF FREQUENCY
$\omega_T = 2\pi f_T$ = FILTER ROLL-OFF TERMINATION FREQUENCY

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$H(\omega)$ can be specified as[4]

$$H(\omega) = \begin{cases} 0 & , |\omega| > \omega_T \quad , \\ 1 & , |\omega| \leq \omega_c \quad , \\ (\omega_T - \omega_c)^{-P} (\omega + \omega_T)^P , & - \omega_T \leq \omega < -\omega_c \quad , \\ (\omega_T - \omega_c)^{-P} (\omega - \omega_T)^P , & \omega_c < \omega \leq \omega_T \quad . \end{cases} \tag{22}$$

In the time domain, the weighting function is given by

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) \, e^{i\omega t} \, d\omega \quad . \tag{23}$$

In terms of the number of weights for a numerical filter, a higher order of decay implies less weights for a given accuracy in the desired frequency representation of the finite filter. The expression for the weights increases rapidly with increasing order of decay. If use is made of the fast Fourier transform (FFT) for the convolution, Eq. (21), large numbers of weights are practical and extremely sharp filter responses can be attained.

For the case of $p = 1$, the weighting function is given as

$$h(t) = \frac{\cos \omega_c t - \cos \omega_T t}{\pi(\omega_T - \omega_c)t^2} \quad . \tag{24}$$

The discrete weights, h_n, can be evaluated directly from $h(t_n)\Delta t$,

[4] J. F. A. Ormsby, "Design of Numerical Filters with Applications to Missile Data Processing," J. ACM, 8, 440-466 (1961).

where equidistant samples are assumed. The assumption that $h(t)$ is real gives $H(-\omega) = H^*(\omega)$, where * denotes the complex conjugate. For zero phase shift, the frequency response function is even or $H(\omega) = |H(\omega)|$ and $h_n = h_{-n}$. In Eq. (21), $u = L = N$, to give a total of $2N + 1$ weights.

For ease of analysis, the frequency will be normalized by the sampling frequency ω_s to give a nondimensional variable $r = \omega/\omega_s$. Equation (24) then becomes

$$h_n = \frac{\cos(2\pi n r_c) - \cos(2\pi n r_T)}{2r_r(\pi n)^2} \quad , \qquad n = 0, \pm 1, \ldots, \pm N \quad , \tag{25}$$

where

$$r_c = \omega_c/\omega_s \quad ,$$

$$r_T = \omega_T/\omega_s \quad ,$$

$$\omega = 2\pi f \quad ,$$

$$r_r = r_T - r_c \quad ,$$

$$t_n = n\Delta t \quad ,$$

$$h_n = h(t_n)\Delta t \quad , \text{ and}$$

$$\Delta t = 2\pi/\omega_s \quad .$$

The relation for the zero order weight is

$$h_o = r_T + r_c \quad . \tag{26}$$

These relations need to be subjected to certain constraints[5] for $\omega_c \to 0$, especially that $H(\omega) = 1$ and $\frac{d^2H(\omega)}{d\omega^2} = 0$ at $\omega = 0$. To

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[5] Ibid.

determine the constrained set of filter weights, the following relations are used:

$$\bar{h}_o = h_o + \frac{\sigma_2 \delta_1 + 2\sigma_1 \delta_2}{\sigma_3} \quad , \tag{27}$$

$$\bar{h}_n = h_n + \frac{\sigma_2 \delta_1 + 2\sigma_1 \delta_2 - n^2 \left[\sigma_1 \delta_1 + (2N + 1)\delta_2\right]}{\sigma_3} \quad , \; n \geq 1 \; , \tag{28}$$

where

$$\delta_1 = 1 - h_o - 2 \sum_{n=1}^{N} h_n \quad ,$$

$$\delta_2 = \sum_{n=1}^{N} n^2 \, h_n \quad ,$$

$$\sigma_1 = \sum_{n=1}^{N} n^2 \quad ,$$

$$\sigma_2 = \sum_{n=1}^{N} n^4 \quad ,$$

$$\sigma_3 = (2N + 1)\sigma_2 - 2\sigma_1^{\,2} \quad .$$

For a high-pass filter with similar characteristics, the weights $(\tilde{h}_n)$ are determined from the low-pass weights $(\bar{h}_n)$ by the relations

$$\tilde{h}_o = 1.0 - \bar{h}_o \quad ,$$

$$\tilde{h}_n = -\bar{h}_n \qquad (29)$$

A band-pass filter can be derived by subtracting the weights of two low-pass filters, but for symmetry a preferable method is frequency shifting. If the low-pass filter frequency function is given by $H(r)$ and weights $\bar{h}_n$, the frequency function for the band-pass filter is defined as $H_B(r) = H(r - r_o) + H(r + r_o)$, where r_o is the center of the desired bandpass. For zero phase shift filters,

$$H_B(r) = 2\bar{h}_o + 2 \sum_{n=1}^{N} (2\bar{h} \cos 2\pi n r_o) \cos 2\pi n r \quad . \qquad (30)$$

The weights for $H_B(r)$ are given by

$$\tilde{h}_n = 2\bar{h}_n \cos (2\pi n r_o) \quad . \qquad (31)$$

Quadrature filters or filters with 90 deg phase shift in the passband can be derived in a similar manner. Once again, it is more expedient to derive a low-pass filter first and then convert to a band-pass or high-pass filter. Accuracies corresponding to the zero phase shift filters can be obtained.

Figure 3 shows the relation between accuracy over the range $0 \leq f \leq f_c$, the number of filter weights, and r_r, a parameter that controls the sharpness of the roll-off. Figure 4 gives the relation between sampling rate, the number of filter weights, and the bandwidth of a band-pass filter. The inherent problem with nonrecursive filters is apparent from these curves--to construct a narrow-band

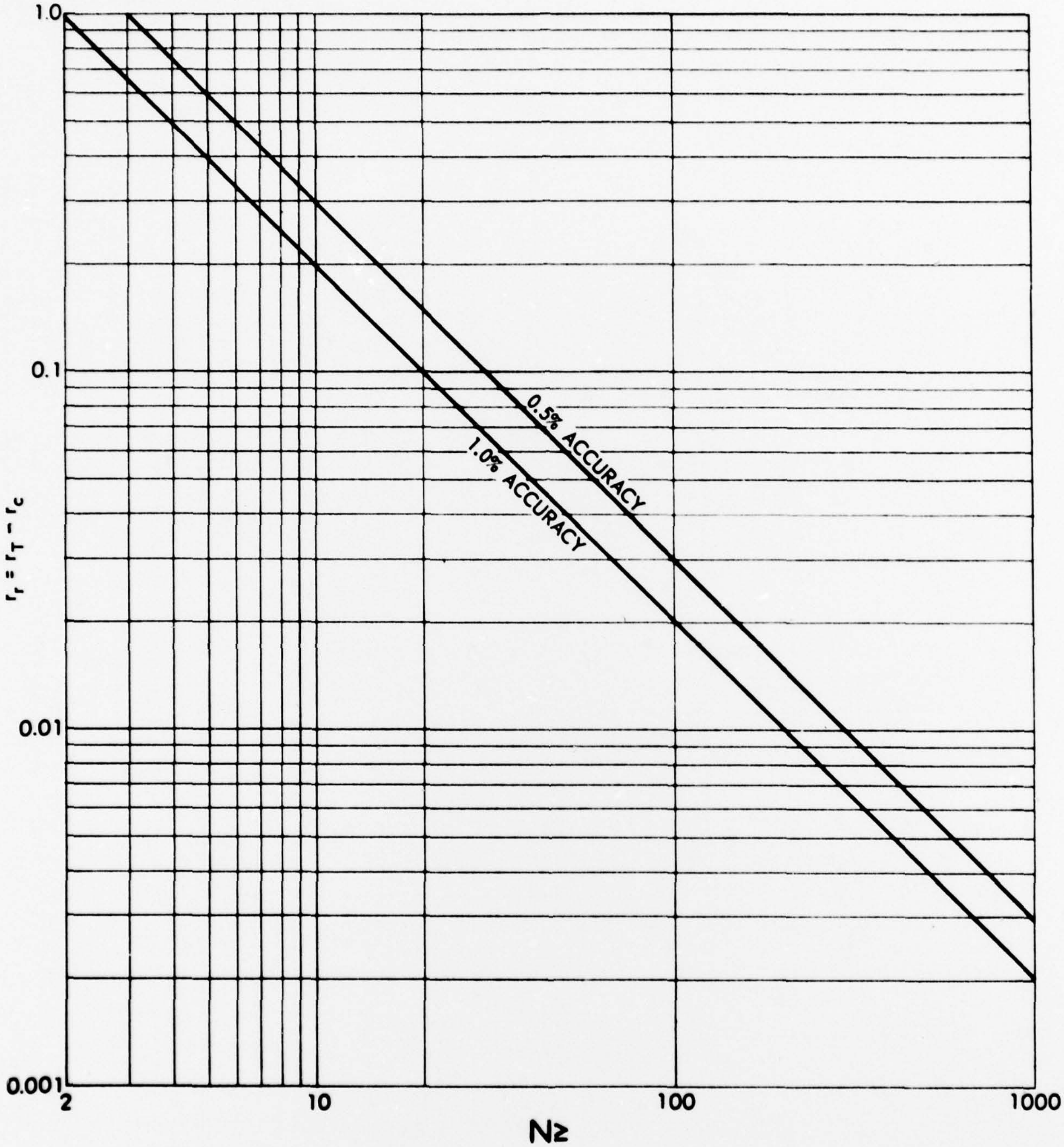

FIGURE 3

NUMBER OF WEIGHTS = (2N + 1)

ACCURACY GIVEN OVER RANGE $0 \leq r \leq r_c$

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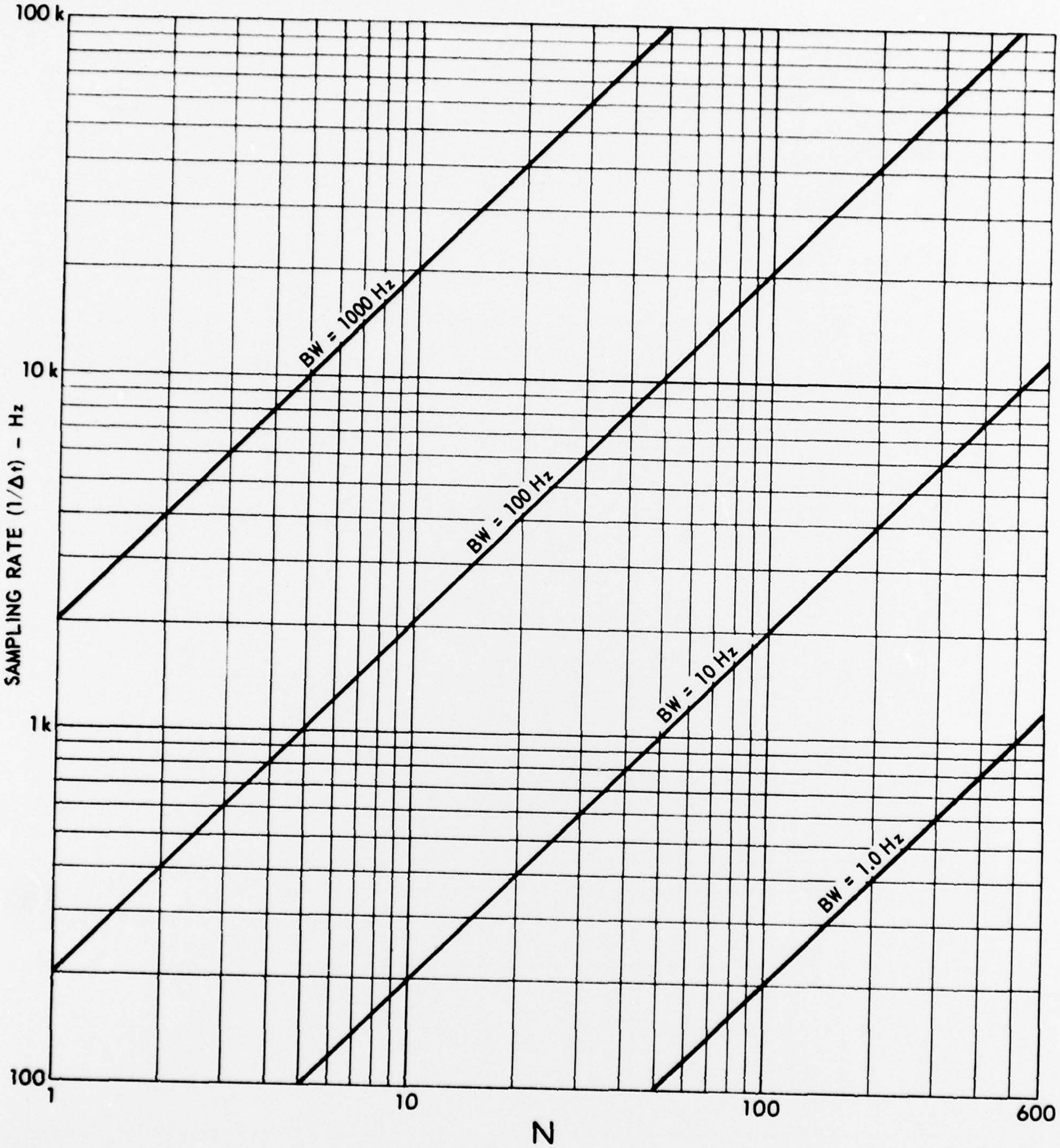

FIGURE 4
NUMBER OF WEIGHTS = (2N + 1)
BW = FILTER BANDWIDTH

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filter at sampling rates near 10 kHz, it is necessary to have a large number of filter weights. This problem can be offset somewhat by using the fast Fourier transform algorithm for performing the convolution with the data series.

B. Recursive Filters

Recursive filtering, or more simply a filter that operates on a data series with a recursive relationship, is analagous to an analog filter with feedback. The digital application of filters of this type is enhanced since a recursive filter normally is derived with only a few weights. The convolution or filtering process does not demand large blocks of computing time as in the case of non-recursive filters.

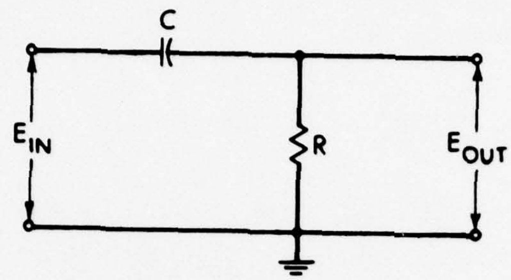
An example of the design of recursive filters with the use of Z-transformations will now be outlined. The filter to be considered is a high-pass RC filter. The circuit and gain characteristics are shown in Fig. 5.

The first step in the digital simulation is to describe the continuous transfer function of the filter by Laplace transform techniques. Since samples from the continuous data are assumed to be taken at discrete intervals of time a, a unit delay is represented by multiplying the continuous Laplace function by e^{-as}, where s is the continuous Laplace variable. The unit delay interval can be expressed in the manner of the Z-transform as

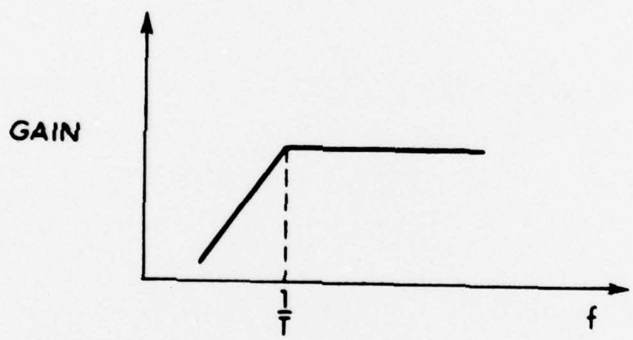
$$Z = e^{-as} = 1 - as + (as)^2/2! - (as)^3/3! + \dots \quad , \qquad (32)$$

or

$$s = -\frac{1}{a}(\log_e Z) \quad . \qquad (33)$$



CIRCUIT OF RC HIGH-PASS FILTER



BODE PLOT OF ABOVE FILTER

FIGURE 5
CHARACTERISTICS OF RC HIGH-PASS FILTER
T = RC

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It is time-consuming to substitute for s in this form. By taking an approximation to Z, the algebra is simplified and the results still match the analog filter results very closely. The approximation for Z is

$$Z = 1 - as + (as)^2/2 - (as)^3/2 \cdot 2 + (as)^4/2 \cdot 2 \cdot 2 - \ldots \quad , \tag{34}$$

which reduces to

$$s = \frac{2}{a}\left[\frac{1 - Z}{1 + Z}\right] \quad . \tag{35}$$

The steps for converting an analog filter to a digital recursive filter by this technique can be summarized as:

1) Put the analog filter transfer function in Laplace transform form;

2) Replace the Laplace s by $\left[\frac{2}{\Delta t}\right]\left[\frac{1 - Z}{1 + Z}\right]$ where Δt is the sampling time of the data series;

3) Reduce the resulting equation to the form

$$E_{out} = \sum_{i=o}^{n} A_i Z^i E_{in} - \sum_{j=1}^{m} B_j Z^j E_{out} \quad ; \tag{36}$$

Interpret Z in Eq. (36) to be a unit sample time delay (Z^2 = two sample times, etc.);

4) Write the equation for the digital filter in the form

$$E_{out}(I) = \sum_{i=o}^{n} A_i\, E_{in}(I - i) - \sum_{j=1}^{m} B_j\, E_{out}(I - j) \quad . \tag{37}$$

For the RC high-pass filter the steps are as follows:

(1) $$\frac{E_{out}}{E_{in}} = \frac{RCs}{1 + RCs} \quad ; \tag{38}$$

(2) $$\frac{E_{out}}{E_{in}} = \frac{A\left[\frac{1 - Z}{1 + Z}\right]}{1 + A\left[\frac{1 - Z}{1 + Z}\right]} \quad , \tag{39}$$

where $A = 2RC/\Delta t$;

(3) $$E_{out} = E_{in}\left[(1 - Z)\frac{A}{1 + A}\right] - E_{out}\left[Z\,\frac{(1 - A)}{1 + A}\right] \quad ; \tag{40}$$

(4) $$E_{out}(I) = \left[E_{in}(I) - E_{in}(I - 1)\right]\left[\frac{A}{1 + A}\right] - E_{out}(I - 1)\left[\frac{1 - A}{1 + A}\right] \quad . \tag{41}$$

Figure 6 shows the frequency response of the RC high-pass filter given by Eq. (41). As in the analog case, the ideal curve cannot be attained. The digital filter simulates the analog filter extremely well, with a -3 dB point at 1/RC or 0.1 Hz, and a slope of -6 dB/octave. If the user wishes a filter with steeper slopes, the filter may be used repeatedly or in cascade. An example of cascading is given in Fig. 7. The filter was used twice, or in the analog sense as a two-stage RC filter, to give a -6 dB point at 1/RC and a -12 dB/octave slope.

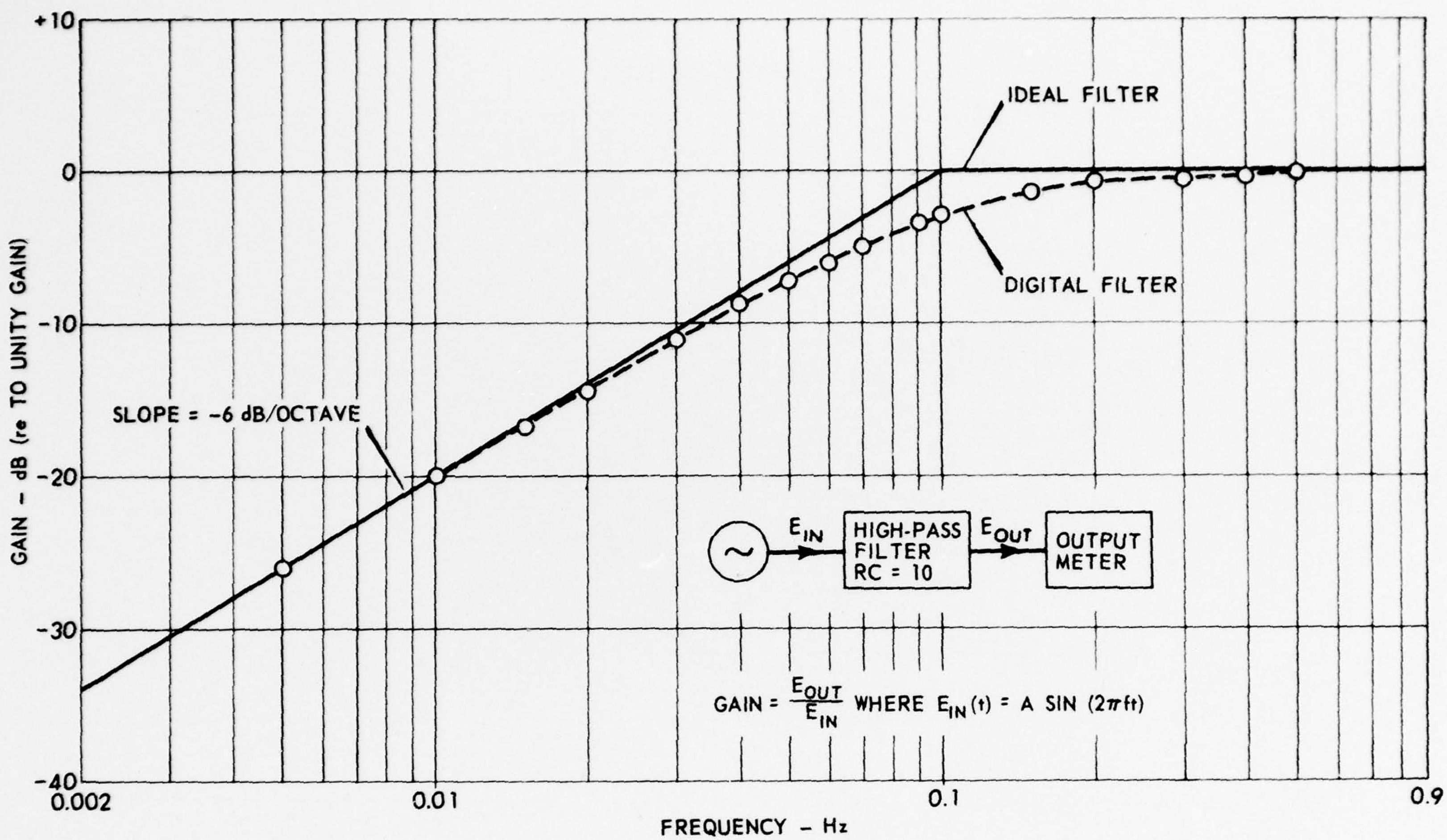

FIGURE 6
RESPONSE OF RECURSIVE MODEL OF RC HIGH-PASS FILTER

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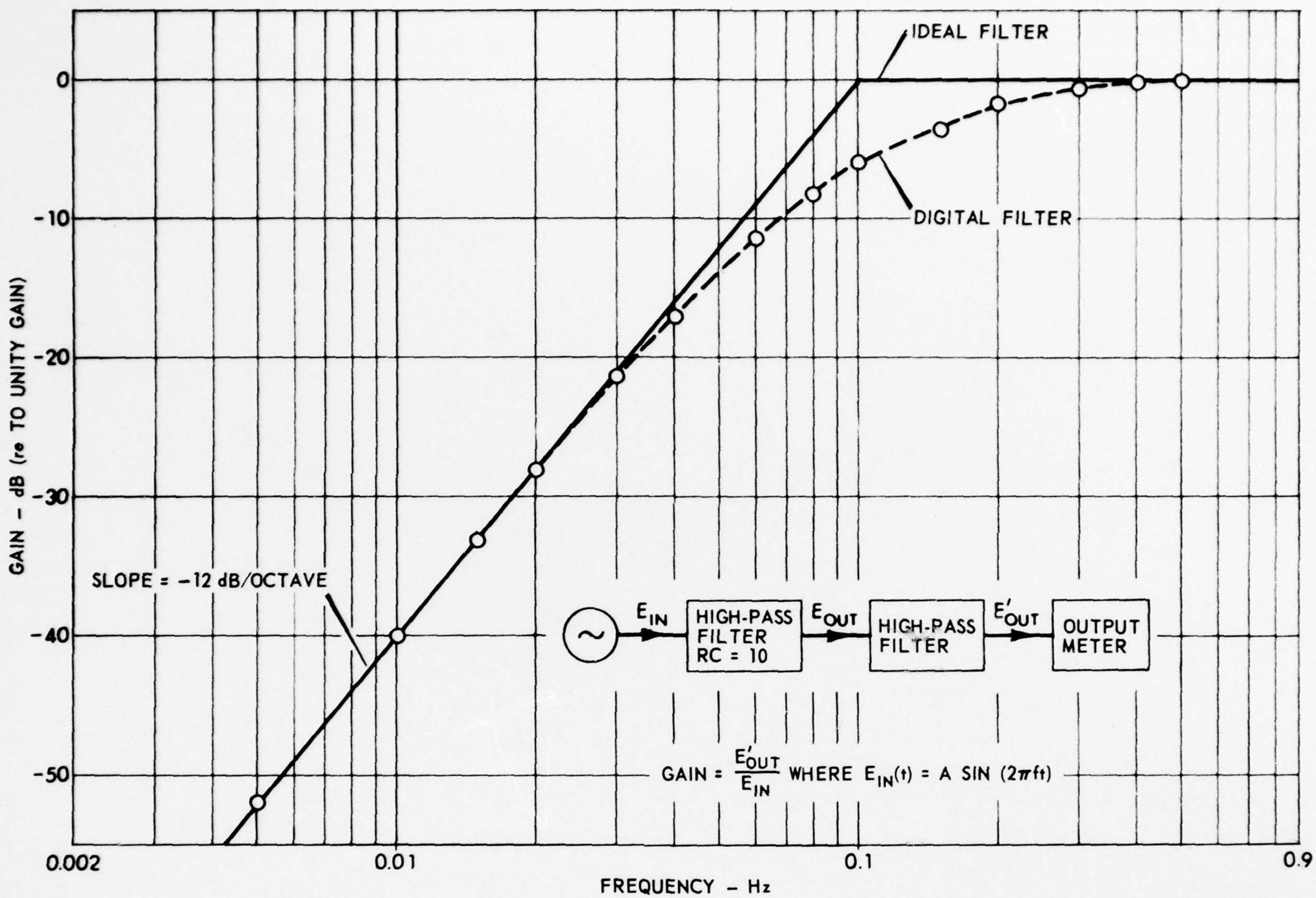

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FIGURE 7
RESPONSE OF CASCADED RC HIGH-PASS FILTER

In terms of computing time, a filter of this type is extremely efficient and fast. It can be cascaded many times for very steep slope without a great expenditure of computer time. The digital simulation of other analog filters can be accomplished in a similar manner. Several very good references on the subject of recursive digital filters are:

1) Blackman, R. B. and Tukey, J. W. The Measurement of Power Spectra (Dover Publication, New York, 1960).

2) Golden, R. M. and Kaiser, J. F. "Design of Wideband Sampled-Data Filters," Bell System Tech. 43, 1533-1646, (1964).

3) Jury, E. I. Theory and Application of the Z-Transform Method (John Wiley and Sons, Inc., New York, 1964).

4) Kuo, F. F. and Kaiser, J. F. System Analysis by Digital Computer (John Wiley and Sons, Inc., New York, 1966), 218-277.

5) Shanks, John L. "Recursion Filters for Digital Processing," Paper presented at the 36th Meeting of the Society of Exploration Geophysicists, Houston, Texas, 1967 (Unpublished).

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III. DIGITAL SIMULATION LANGUAGES
(G. E. Ellis)

The discrete simulation language GASP II has been implemented on the CDC 3200 computer. GASP (General Activity Simulation Program) consists of thirteen subroutines and functions written in FORTRAN and specifically designed for use in simulation studies of industrial systems.

GASP II provides the programmer with a library of subprograms which may be used to accomplish certain operations common to many simulation studies and models. In general, GASP will perform the following functions: start the simulation, move the model through time, stop the simulation, monitor and present intermediate results of the simulation if desired, provide random variables drawn from the more common probability distributions according to parameters specified by the programmer, collect and classify data, compute statistics, detect and isolate errors in the program, and present a summary report of the simulation.

The present version of GASP II was written for a computer with a limited version of FORTRAN and a small memory. This quarter has been devoted to updating the coding with the more flexible FORTRAN available for the CDC 3200 and expanding the program size to fit the 32k memory bank of the 3200.

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IV. PERT
(L. A. Noack, G. E. Ellis)

A model research program was devised to include experimental, theoretical, and data reduction phases. The program, to be conducted during a one-year period, is shown in Fig. 8. The network is arranged in a normal PERT/TIME configuration. The critical path, or longest path in time, is shown as derived from the computer output from the CDC 3200.

At first glance, the outstanding disadvantage that appears in the network is the lack of a feedback loop, which is essential in all research programs. Using the standard PERT/TIME computer programs, the feedback loop can be implemented by the scientist and the computer, updating with new projected times and/or a modification of the network.

The simulation language GASP II will be used in the next quarter to implement PERT/TIME techniques. If this is successful, a feedback loop in the process will be implemented using GASP II to more closely model the actual research process.

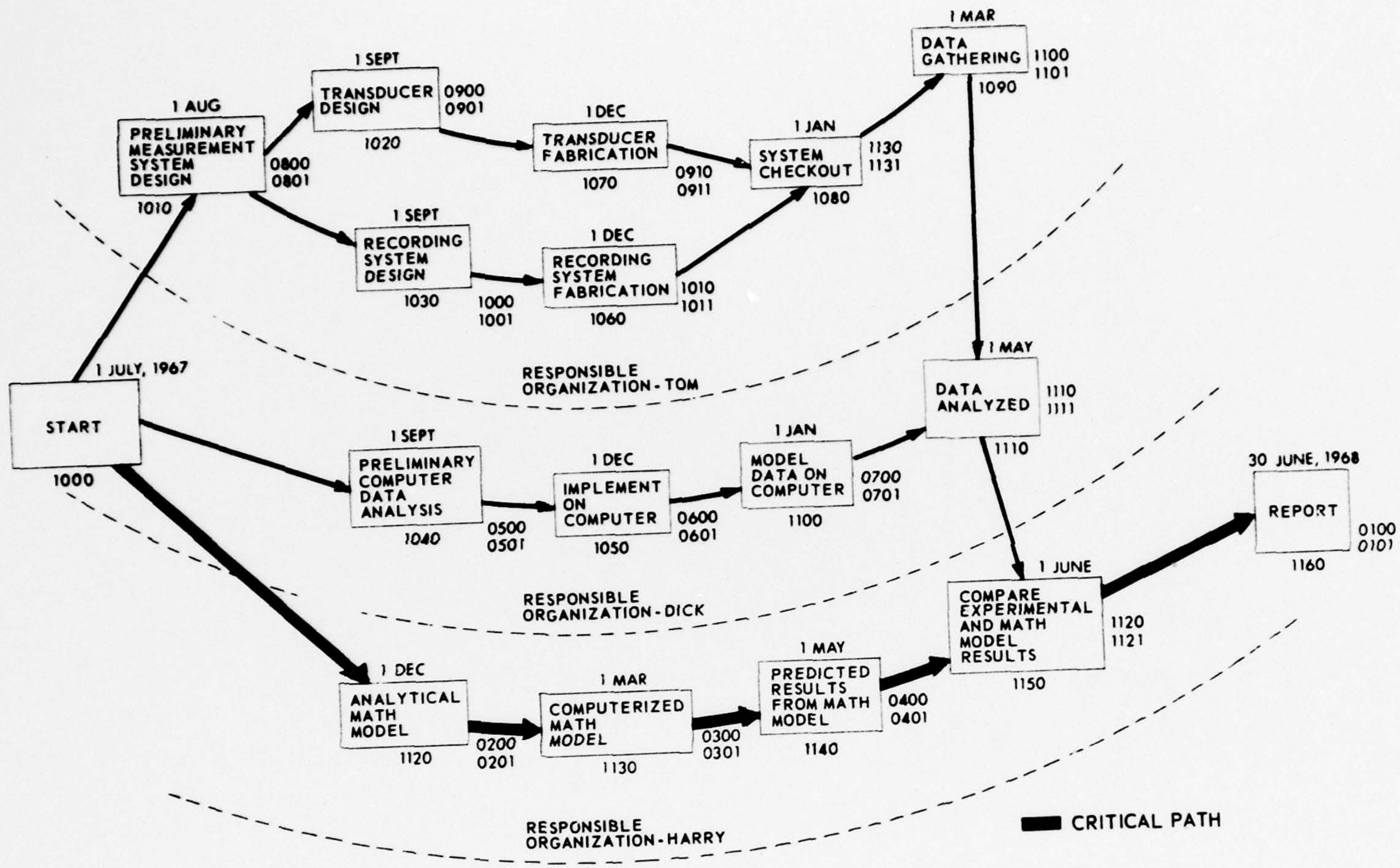

FIGURE 8
PERT/TIME

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